terms of n other quantities $b_1 \ldots\ldots b_n$, then is the space of the first quantities identical with that of the last quantities. But if the n quantities $a_1 \ldots\ldots a_n$ can be expressed in terms of less than n quantities $b_1 \ldots\ldots b_n$, then $a_1 \ldots\ldots a_n$ are not independent, and some of them can be numerically expressed in terms of others.

20. *Two quantities of a space of the nth order are equal to each other when and only when their numerical coefficients of the same units are equal.* This is analogous to the algebraic theorem which says that two complex numbers are equal only when their real parts are equal and also their imaginary parts.

21. *If the coefficients $x_1 \ldots\ldots x_n$ by which an extensive quantity x is expressed in terms of the units $e_1 \ldots\ldots e_n$ satisfy an equation of the mth degree $f(x_1 \ldots\ldots x_n)=0$, then the coefficients $y_1 \ldots\ldots y_n$ by which x is expressed in terms of $a_1 \ldots\ldots a_n$ of the same space also satisfy an equation of the mth degree, and if the first equation is homogeneous, the latter is also.*

PROOF. Let $a_1 = \Sigma \alpha_{1r} e_r$, ... Then we have

$$x_1 e_1 + x_2 e_2 + \ldots + y_1 \Sigma \alpha_{1r} e_r + y_2 \Sigma \alpha_{2r} e_r + \ldots = \Sigma y_r \alpha_{r1} . e_1 + \Sigma y_r \alpha_{r2} . e_2 + \ldots$$

$$\therefore\ x_1 = \Sigma y_r \alpha_{r1},\ \ x_2 = \Sigma y_r \alpha_{r2},\ \ldots\ldots \quad \text{(Art. 20)}.$$

But if these values are substituted in $f(x_1, \ldots\ldots x_n)=0$, we get an equation of the mth degree in $y_1, y_2, \ldots\ldots$, and, indeed, homogeneous if the first equation is homogeneous.

[To be Continued.]

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

112. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

The cost of an article is $\$4.\frac{.297}{1.00\dot{2}}$. The selling price is $\$6.\frac{1000}{33337}$. What is the gain %?

I. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville Tenn., and the PROPOSER.

$297 = \frac{297}{999} = \frac{11}{37}$; $1.003 = 1\frac{1}{300} = \frac{301}{300}$.

$\frac{11}{37} \div \frac{301}{300} = \frac{3300}{11137}$; $.\frac{3300}{11137} = \frac{330}{11137}$.

$\therefore\ \$4.\dfrac{\dot{2}9\dot{7}}{1.00\dot{3}}=\$4\frac{330}{11137}=\$\frac{44878}{11137}=$cost price.

$\$6.\frac{1000}{33337}=\$6\frac{100}{33337}=\$\frac{200122}{33337}=$selling price.

$\therefore\ \$\frac{200122}{33337}-\$\frac{44878}{11137}=\$\frac{19801644}{100344437}=$gain.

$(\frac{1980164 4}{100344437}\div\frac{44878}{11137})$ of $100\%=48\frac{19640328}{20217539}\%$.

Also solved by *O. S. WESTCOTT* and *ALOIS F. KOVARIK.*

113. Proposed by B. F. SINE, Principal of Normal School, Capon Bridge, W. Va.

In what time can a note of \$5280, bearing 6% interest, be paid by paying \$600 a year? [Solve by arithmetic].

Solution by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

Let Q be the principal, r the rate, n the number of years, and P annual payment. Then

$Q(1+r)=$amount due at end of first year.

$Q(1+r)-P=$principal to run second year.

$Q(1+r)^2-P(1+r)=$amount due at end of second year.

$Q(1+r)^2-P(1+r)-P=$principal to run third year.

....

$Q(1+r)^n-P(1+r)^{n-1}\ldots\ldots P(1+r)-P=$amount to run $(n+1)$th year.

But the debt is cancelled. Hence

$$Q(1+r)^n-P(1+r)^{n-1}-\ldots\ldots P(1+r)-P=0.$$

$$\therefore P\left[\frac{(1+r)^n-1}{r}\right]=Q(1+r)^n.$$

$$\therefore (P-Qr)(1+r)^n=P.$$

$$\therefore n=\frac{\log P-\log(P-Qr)}{\log(1+r)}.$$

In the problem $P=\$600$, $Q=\$5280$, $r=.06$.

$$\therefore n=\frac{\log 600-\log(600-5280\times.06)}{\log(1.06)}=12.88 \text{ years}.$$

Also solved by *G. B. M. ZERR*, *COOPER D. SCHMITT*, and *J. SCHEFFER.*

ALGEBRA.

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93. Proposed by CHARLES C. CROSS, Whaleyville, Va.

Given $x^x+y^y=285$, and $y^x-x^y=14$, to find the values of x and y. [From *Bonnycastle's Algebra*, 1841].

I. Solution by A. H. BELL, Hillsboro, Ill.

The two equations give

$$(285-y^y)^y=(y^x-14)^x\ldots\ldots(1).$$